0368-4283 – Space-Bounded Computation

Wednesdays, 14:00-17:00.

Grading policy:

- Project – 70%. A reading project.
- Homework – 15%.
- Participation in class – 15%.
- Bonuses for help in forum and taking scribe.

Extended Syllabus

Part I – The basics		
Basic Classes. Some representative problems.	The classes $\mathsf{DSpace}(s(n))$, $\mathsf{BPSpace}(s(n))$, $\mathsf{RSpace}(s(n))$ and $\mathsf{NSpace}(s(n))$.	[AB]
	Circuit classes: NC^k, AC^k and NC.	[AB]
	Some languages (and problems) we want to classify: addition, multiplication, parity, majority, sorting, undircted connectivity, directed connectivity (STCON), Det, Perm, Maximal Independent Set (MIS), Perfect Matching (PM), Polynomial identity testing (PIT).	
Part II – Random Walks over Undirected graphs		
From Combinatorics to Algebra	Undirected graphs as operators.	
	Spectral gap and rapid mixing.	
	Undirected connectivity is in RL.	[AK+], [AS]
	Universal traversal sequences and universal exploration sequences.	
A random walk over an expander as a replacement to independent random samples	Expanders: Graphs with a large spectral gap.	
	The expander mixing lemma.	[V]
	The expander Chernoff bound.	[H]
	Bias samplers.	
	A comparison with other samplers (extractors, condensers, and more).	

Part III – The Zig-Zag product and its ramifications for Space Bounded computation		
Growing a graph into an expander	Cayley graphs, Abelian Cayley graphs, Explicit exapnders with logarithmic degree.	
	The art of turning big problems to small: The Zig-Zag product.	[RVW]
	A "combinatorial" construction of fully explicit expanders.	[RVW]
	Undirected Connectivity is in Deterministic LogSpace.	[R]
	Explicit universal exploration sequences	[RVW]
Inverting the Laplacian in small space	The Laplacian of undirected graphs	[L]
	Sparsifiers. Sparsifying the clique: Derandomized Squaring	[MR+]
	Approximating the inverse of the Laplacian of an undirected graph	[MR+]
	Richardson iteration	[MR+]

Part IV – Pseudo-random Generators		
Nisan's generator	Branching programs: The non-uniform analogue of BPL.	
	Pseudo-Randomness.	
	Nisan's generator (with hash functions and pair-wise independence; with expanders and the expander mixing lemma; with extractors).	[N1,NZ]
	The INW generator. Curving the seed from the inside, or taking it for each level from the outside.	[INW]
	$\mathsf{BPL} \subseteq \mathsf{DTimeSpace}(\mathrm{poly}(n), \log^2 n)$	[N2]
RL is in $\mathsf{DSpace}(\log^{1.5} n)$	Pseudo-deterministic algorithms. Consistent sampling using shift and truncate.	[SZ]
	The Saks and Zhou derandomization algorithm.	[SZ]
PRGs against more restricted adversaries	A PRG against combinatorial rectangles.	
	A PRG against regular branching programs.	
	A PRG against half spaces.	

References

[V]	Salil Vadhan	Pseudorandomness (link)
[LW]	Michael Luby and Avi Wigderson	Pairwise Independence and Derandomization (link)
[L]	László Lovász	Random Walks on Graphs: A Survey (link)
[AB]	Sanjeev Arora and Boaz Barak	Computational Complexity: A Modern Approach
[AK+]	R. Aleliunas, R. M. Karp, R. J. Lipton, L. Lovász and C. Rackof	Random walks, universal traversal sequences, and the complexity of maze problems
[AS]	Noga Alon and Benny Sudakov	Bipartite subgraphs and the smallest eigenvalue (link)
[H]	Alexander Healy	Randomness-Efficient Sampling within NC^1 (link)
[RVW]	Omer Reingold, Salil Vadhan and Avi Wigderson	Entropy waves, the zig-zag graph product, and new constant-degree (link)
[R]	Omer Reingold	Undirected Connectivity in Log-Space (link)
[MR+]	J. Murtagh, O. Reingold, A. Sidford and S. Vadhan	Derandomization Beyond Connectivity: Undirected Laplacian Systems in Nearly Logarithmic Space (link)
[N1]	Noam Nisan	Pseudorandom generators for space-bounded computation (link)
[N2]	Noam Nisan	$\mathsf{RL} \subseteq \mathsf{SC}$ (link)
[INW]	Russel Impagliazzo, Noam Nisan and Avi Wigderson	Pseudorandomness for Network Algorithms (link)
[NZ]	Noam Nisan and David Zuckerman	Randomness is linear in space (link)
[SZ]	Michael Saks and Shiyu Zhou	$\mathsf{BP_H SPACE}(S) \subseteq \mathsf{DSPACE}(S^{3/2})$ (link)